A Derivation of $\frac{\partial T}{\partial E}$

The solution to the maximization problem in (2) is denoted by $T^*(w, \overline{Y}, E)$. To evaluate how T^* changes with an improvement in E I implicitly differentiate the first order condition in (3) with respect to E. After rearrangement, I obtain

$$\frac{\partial T^*}{\partial E} = \underbrace{\frac{1}{\Delta}\left\{(U_{HH}\frac{\partial H}{\partial T} - wU_{ZH})\frac{\partial H}{\partial E}\right\}}_{\text{income effect}} + \underbrace{U_H\frac{\partial^2 H}{\partial E \partial T}}_{\text{substitution effect}}$$

where

$$\Delta = -(\frac{\partial H}{\partial T})^2 U_{HH} + 2wU_{HZ}\frac{\partial H}{\partial T} - w^2 U_{ZZ} - U_H\frac{\partial^2 H}{\partial T^2}$$

First, consider the sign on Δ. The quasiconcavity of the utility function implies that first three terms are non-negative and the concavity of H implies that the last term is non-negative. Therefore, Δ is non-negative. Now, I describe the sufficient conditions for $\frac{\partial T^*}{\partial E} \leq 0$,

- $(U_{HH}\frac{\partial H}{\partial T} - wU_{ZH})\frac{\partial H}{\partial E} \leq 0$ which given $\frac{\partial H}{\partial E} > 0$, is equivalent to requiring $U_{HH}\frac{\partial H}{\partial T} - wU_{ZH} \leq 0$ which is equivalent to Z being normal.[1]

- $U_H\frac{\partial^2 H}{\partial E \partial T} \leq 0$ which given $U_H > 0$ requires $\frac{\partial^2 H}{\partial E \partial T} \leq 0$, i.e. that the marginal impact of treatment on health be decreasing in water quality.

- If $U_H\frac{\partial^2 H}{\partial E \partial T} \geq 0$ which given $U_H > 0$ requires $\frac{\partial^2 H}{\partial E \partial T} \geq 0$, (i.e. that the marginal impact of treatment on health is increasing in water quality), then we could still have a reduction in T as long as the income effect dominates the substitution effect.

[1]To see why, derive the comparative static $\frac{d\overline{Z}}{d\overline{Y}}$, where $\overline{Z}$ denotes the Walrasian demand for Z. Note that $w/\frac{\partial H}{\partial T}$ is the shadow price of health in equilibrium.